



University of Sri Jayewardenepura
Faculty of Humanities and Social Sciences

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Bachelor of Arts Second Year First Semester Examination
August/September - 2018

Economics

ECON 2150.3 - Mathematical Economics

Time allowed: Three hours (03)

Answer any five (05) questions. Calculators are allowed to use.
Equal marks will be given for each question. 80 marks allocated for the final exam
and 20 marks for the continuous assessment.

1. i. Find the derivatives of the following functions using appropriate rules.

a) $(5x^3 + x)(x^5 - 3x^3 - 4)$

b) $y = 8e^{3x} - 3e^{-4x^2}$

c) $y = \ln\left(\frac{2x^2 - 5x + 4}{x^2 + 3x}\right)$

d) $y = (3x^3 + 2x + 5)^2$

e) $y = \frac{x^2 + 3x + 2}{\sqrt{x}}$

(10 Marks)

ii. Find the critical points of the following function may be optimized and determine whether at these points the function is maximized, is minimized, is at inflection point or is at saddle point.

a) $z = 3x^2 - xy + 2y^2 - 4x - 7y + 12$

b) $z = 60x + 34y - 4xy - 6x^2 - 3 - y^2 + 5$

c) $z = 3x^3 - 9xy - 3y^3$

(06 Marks)

2. i. Determine the marginal revenue (MR) / marginal cost (MC) function for the following functions;

(a) $AVC = 10Q + 4$ $TFC = 50$

(b) $TR = 1400Q - 7.5Q^2$

(c) $TC = 35 + 5Q - 2Q^2 + 2Q^3$

(d) $P = Q^2 + 2Q + 1$

(e) $AC = 1.5Q + 4 + \frac{46}{Q}$

(10 Marks)

ii. If the utility function of an individual takes the form

$$U = U(x_1, x_2) = (x_1 + 2)^2(x_2 + 3)^3$$

Where U is total utility, and x_1 and x_2 are the quantities of two commodities consumed:

- Find the marginal-utility function for each of the two commodities.
- Find the value of the marginal utility of the first commodity when 3 units of each commodity are consumed.

(06 Marks)

03 i. If $z = 3x^3 - 5y^2 - 225x + 70y + 23$,

Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$, $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$ (06 Marks)

- ii. A monopolistic firm has the following demand functions for each of its product x and y.

$$X = 72 - 0.5P_x$$

$$Y = 120 - P_y$$

The combined cost function is $c = x^2 + xy + y^2 + 35$, and maximum joint production is 40. Thus $x + y = 40$. find the profit maximizing level of

- (a) output (b) price and (c) maximum profit. (10 Marks)

- 04 i. A producer has a possibility of applying price discrimination in domestic and foreign markets for a product. The demand curves for the products are,

$$Q_1 = 21 - 0.1P_1$$

$$Q_2 = 50 - 0.4P_2$$

And the joint cost function $C = 2000 + 10Q_1 + 10Q_2$

- (a) Find the profit maximizing price (P_1) and output (Q_1) at the domestic market.
 (b) Find the profit maximizing price (P_2) and output (Q_2) at the foreign market.
 (c) If there is no price discrimination in two markets ($Q = Q_1 + Q_2$), find the profit maximizing price and output

(10 Marks)

- ii. Consumer utility function is given by

$$U = Q_1 Q_2$$

Prices of Q_1 and Q_2 are given as Rs.1/= and Rs.4/= respectively. Consumer income is Rs. 120/=.

- (a) Maximize the above utility function subject to the given budget constraint.
 (b) What happens to the new optimal utility if the consumer income rises by Rs.1/=.

(06 Marks)

- 05 i. Find the following indefinite integrals.

a. $\int (24x^5 + 35x^4 - 64x^3) dx$

b. $\int 9\sqrt{x-13} dx$

(06 Marks)

- ii. Evaluate each of the following definite integrals

a. $\int_2^5 (8x + 7) dx$

b. $\int_8^{125} (4x^{-\frac{1}{3}} - x^{-\frac{2}{3}}) dx$

(06 Marks)

- ii. Given the demand function $P_d = 30 - Q^2$ and supply function $P_s = 2Q + 6$. Assuming pure competition, find

- a. the consumer's surplus
 b. the producer's surplus

(04 Marks)

06 i. Determine the equilibrium price of three independent commodities using matrix inverse method

$$\begin{aligned}2P_1 + 4P_2 - 3P_3 &= 12 \\3P_1 - 5P_2 + 2P_3 &= 13 \\-P_1 + 3P_2 + 2P_3 &= 17\end{aligned}$$

(08 Marks)

ii. Use Cramer's rule and Lagrangian multiplier to optimize the following functions subject to given constraint,

$$Z = 80x - 2x^2 - xy - 3y^2 + 100y \quad \text{Subject to } x + y = 12$$

(08 Marks)

07 i. Consider following matrices ,

$$A = \begin{bmatrix} 7 & 11 \\ 2 & 9 \\ 10 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 12 & 4 \\ 3 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 2 & 5 \\ 7 & 9 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 5 \\ 1 \\ 10 \end{bmatrix}$$

- Find the determinants of A and B matrices.
- Find BC, AC and CD

(06 Marks)

ii. A firm operates at perfectly competitive market produces two goods and has the total revenue and total cost functions;

$$TR = 18Q_1 - Q_2 = 87$$

$$TC = -2Q_1 + 36Q_2 = 98$$

- Find the profit maximizing output
- Use Cramer rule to prove that firm earns a maximum profit.

(10 Marks)
