University of Sri Jayewardenepura
Faculty of Humanities and Social Sciences

Bachelor of Arts Second Year First Semester Examination
July /August- 2017
Economics
ECON 2150.03 – Mathematical Economics

Time allowed: Three hours (03)

Answer any five (05) questions. Calculators are allowed to use.
Equal marks will be given for each question. 80 marks allocated for the final exam and 20 marks for the continuous assessment.

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1. (i.) Find the derivatives of the following functions applying appropriate rules.

(a) $Y = \frac{(3x+4x^2)}{(2x+6x)(x^4-5x)}$

(b) $Y = e^{(2x+3)}\ (x^3 + 4x)$

(10 Marks)

(ii.) The demand function and the average cost function of a firm is given below.

$P = 24 - 2Q$

$AC = 4Q + 6 + \frac{16}{Q}$.

(a)Set up the profit function

(b) Find the profit maximizing output

(c) Confirm that firm has a maximum profit when MR=MC

(06 Marks)

2. (i.) Find the first (f_x, f_y) and second (f_{xx}, f_{yy}) partial deravatives of the following funcitions.

(a) $Z = \left(\frac{8x+7y}{5x+2y}\right)^2$ (b)$Z=(5x^2 - 4y)^2\ (2x + 7y^3)$

(10 Marks)

(ii.) A monopolistic competition producer offers two different products; x, y for which the demand functions are,

$$Q_x = 14 - 0.25P_x$$

$$Q_y = 24 - 0.5P_y$$

The combined cost function is,

$$TC = {Q_x}^2 + 5Q_xQ_y + {Q_y}^2$$

Find the profit maximizing level of,

(a) Output

(b) Price

(c) Profit *(06 marks)*

3. (i.) Find the critical values of following functions and determine whether the function is minimized, maximized at the points or determine whether it is an a saddle point.

(a) $z = 3x^2 - xy + 2y^2 - 4x - 7y + 12$

(b) $z = x^3 - 6x^2 + 2y^3 + 9y^2 - 63x - 60y$

(c) $z = 3x^3 + 1.5y^2 - 18xy + 17$

(06 Marks)

(ii.) Maximize following utility functions subject to budget constraint using Lagrange method.

(a) $u = q_1q_2$ $P_1 = 1, P_2 = 4$ and budget B= 120

(b) $u = q_1q_2 + q_1 + 2q_2$ $P_1 = 2, P_2 = 5$ and budget B= 51

(04 Marks)

(iii). Optimized following Cobb-Douglus production functions subject to the given constaint using Lagrange method.

(a) $q = K^{.03}L^{0.5}$ Subject to 6K+2L= 384

(b) $q = 10K^{0.7}L^{0.3}$ $P_k = 28$, $P_L = 10$ and B = 4000

(06 Marks)

4. (i). Find the integral of the following functions,

(a) $\int \frac{1}{x^6} dx$

(b) $\int 4x^2(6x^3 + 1)dx$

(c) $\int (6x - 22)^{-6} dx$

(d) $\int_1^3 (4x^3 + 3x)\, dx$

(e) $\int_{-4}^5 (8x^3 + 9x^2)dx$

(10 Marks)

(ii.) Given the demand function $Pd = 25 - Q_d^2$ and supply function $Ps = 2Qs + 1$,

If perfect competition exists, find;

(a) The consumer surplus
(b) The producer surplus *(06 Marks)*

5. (i.) Use the metrix inverse method to find the equilibrium price of the following commodities which are independent to each other.

$$11P_1 - P_2 - P_3 = 31$$
$$-P_1 + 6P_2 - 2P_3 = 26$$
$$-P_1 - 2P_2 + 7P_3 = 24$$

(08 Marks)

(ii.) Use Cramer's rule and Lagrangian multiplier to optimize the following functions subject to given constraint,

$Z = 5x^2 - 2xy + 3y^2 + 800$ Subject to $x + y = 39$

(08 Marks)

6. (i). Consider following Matrices ,

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 5 & 1 \\ 1 & 0 & 3 \\ 4 & -1 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 4 \end{bmatrix}$$

(a) Find the determinants of A and B matrices.

(b) Find BC, AC and CD

(06 Marks)

(ii.) Total revenue and the total cost functions of a perfectly competitive firm who produces two goods are given below,

$$TR = 15Q_1 + 18Q_2$$
$$TC = 2{Q_1}^2 + 2Q_1Q_2 + 3{Q_2}^2$$

(a) Find the profit maximize output

(b) Use Hessian matrix to prove that the firm has a maximum profit.

(10 Marks)

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